



CANDIDATE  
NAME

--

CENTRE  
NUMBER

--	--	--	--	--

CANDIDATE  
NUMBER

--	--	--	--

## 0606/13

May/June 2022

**2 hours**

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series*      $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series*      $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

**2. TRIGONOMETRY***Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

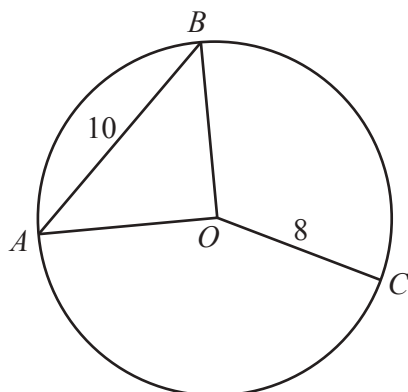
*Formulae for  $\triangle ABC$* 

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Find the rational numbers  $a$ ,  $b$  and  $c$ , such that the first three terms, in descending powers of  $x$ , in the expansion of  $\left(3x^2 - \frac{1}{9x}\right)^5$  can be written in the form  $ax^{10} + bx^7 + cx^4$ . [3]

- (b) Hence find the coefficient of  $x^4$  in the expansion of  $\left(3x^2 - \frac{1}{9x}\right)^5 \left(1 + \frac{1}{x^3}\right)^2$ . [3]

- 2 In this question, all lengths are in centimetres and all angles are in radians.



The diagram shows a circle, centre  $O$ , radius 8. The points  $A$ ,  $B$  and  $C$  lie on the circumference of the circle. The chord  $AB$  has length 10.

- (a) Show that angle  $BOA$  is 1.35 correct to 2 decimal places. [2]

- (b) Given that the minor arc  $BC$  has a length of 18, find angle  $BOC$ . [2]

- (c) Find the area of the minor sector  $AOC$ . [3]

- 3 (a) Find the exact solution of the equation  $2e^{6x} - 3e^{3x} - 5 = 0$ . [3]

- (b) Solve the following simultaneous equations.

$$e^{4x-7} \div e^{5x+7y} = \frac{1}{e^2}$$

$$xy + 18 = 0$$
 [5]

- 4 Variables  $x$  and  $y$  are such that when  $e^{4y}$  is plotted against  $x$ , a straight line of gradient  $\frac{2}{5}$ , passing through  $(10, 2)$ , is obtained.

(a) Find  $y$  in terms of  $x$ . [3]

(b) Find the value of  $y$  when  $x = 45$ , giving your answer in the form  $\ln p$ . [2]

(c) Find the values of  $x$  for which  $y$  can be defined. [1]

5 The velocity,  $v \text{ ms}^{-1}$ , of a particle moving in a straight line,  $t$  seconds after passing through a fixed point  $O$ , is given by  $v = 6 \sin 3t$ .

(a) Find the time at which the acceleration of the particle is first equal to  $-9 \text{ ms}^{-2}$ . [4]

(b) Find the displacement of the particle from  $O$  when  $t = 5.6$ . [4]

6 (a) It is given that  $f : x \rightarrow 2x^2$  for  $x \geq 0$ ,

$$g : x \rightarrow 2x + 1 \text{ for } x \geq 0.$$

Each of the expressions in the table can be written as one of the following.

$$f' \quad f'' \quad g' \quad g'' \quad fg \quad gf \quad f^2 \quad g^2 \quad f^{-1} \quad g^{-1}$$

Complete the table. The first row has been completed for you.

[5]

Expression	Function notation
2	$g'$
0	
$4x$	
$8x^2 + 8x + 2$	
$4x + 3$	
$\frac{x-1}{2}$	



(b) It is given that  $h(x) = (x-1)^2 + 3$  for  $x \geq a$ . The value of  $a$  is as small as possible such that  $h^{-1}$  exists.

(i) Write down the value of  $a$ . [1]

(ii) Write down the range of  $h$ . [1]

(iii) Find  $h^{-1}(x)$  and state its domain. [3]

7 A curve has equation  $y = \frac{(2x+1)^{\frac{3}{2}}}{x+5}$  for  $x \geq 0$ .

(a) Show that  $\frac{dy}{dx} = \frac{(2x+1)^{\frac{1}{2}}}{(x+5)^2}(Ax+B)$ , where  $A$  and  $B$  are integers to be found. [4]

(b) Show that there are no stationary points on this curve. [1]

(c) Find the approximate change in  $y$  when  $x$  increases from 1 to  $1 + p$ , where  $p$  is small. [2]

(d) Given that when  $x = 1$  the rate of change in  $x$  is 2.5 units per second, find the corresponding rate of change in  $y$ . [2]

- 8 (a)** A 6-digit number is formed from the digits 0, 1, 2, 5, 6, 7, 8, 9. A number cannot start with 0 and each digit can be used at most once in any 6-digit number.
- (i)** Find how many 6-digit numbers can be formed if there are no further restrictions. [1]
- (ii)** Find how many of these 6-digit numbers are divisible by 5. [3]
- (iii)** Find how many of these 6-digit numbers are greater than 850 000. [3]

- (b) A team of 8 people is to be chosen from 12 people. Three of the people are brothers who must not be separated. Find the number of different teams that can be chosen. [3]

- 9 (a) Solve the equation  $3 \operatorname{cosec}^2\left(2\phi - \frac{\pi}{3}\right) = 4$ , for  $0 < \phi < \pi$ . Give your solutions in terms of  $\pi$ .  
[4]

- (b) Given that  $2x - 1 = \operatorname{cosec}^2\theta$  and  $y + 1 = \tan^2\theta$ , find  $y$  in terms of  $x$ .  
[4]

**10 (a)** Show that  $\frac{6}{2+3x} + \frac{4}{(x+1)^2} - \frac{2}{x+1}$  can be written as  $\frac{14x+10}{(2+3x)(x+1)^2}$ . [2]

**(b)** Hence find the exact value of  $\int_0^2 \frac{14x+10}{(2+3x)(x+1)^2} dx$ . Give your answer in the form  $p + \ln q$ , where  $p$  and  $q$  are rational numbers. [6]