## Cambridge IGCSE ${ }^{\text {TM }}$

CANDIDATE NAME

CENTRE


## ADDITIONAL MATHEMATICS

Paper 1
May/June 2022

You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].


## Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n}
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$

Arithmetic series

$$
\begin{aligned}
& u_{n}=a+(n-1) d \\
& S_{n}=\frac{1}{2} n(a+l)=\frac{1}{2} n\{2 a+(n-1) d\}
\end{aligned}
$$

Geometric series

$$
\begin{aligned}
& u_{n}=a r^{n-1} \\
& S_{n}=\frac{a\left(1-r^{n}\right)}{1-r} \quad(r \neq 1) \\
& S_{\infty}=\frac{a}{1-r}(|r|<1)
\end{aligned}
$$

## 2. TRIGONOMETRY

## Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1 Find constants $a, b$ and $c$ such that $\frac{\sqrt{p} q^{\frac{2}{3}} r^{-3}}{\left(p q^{-1}\right)^{2} r^{-1}}=p^{a} q^{b} r^{c}$.

2 A particle moves in a straight line such that its displacement, $s$ metres, from a fixed point, at time $t$ seconds, $t \geqslant 0$, is given by $s=(1+3 t)^{-\frac{1}{2}}$.
(a) Find the exact speed of the particle when $t=1$.
(b) Show that the acceleration of the particle will never be zero.

3 A function f is such that $\mathrm{f}(x)=\ln (2 x+1)$, for $x>-\frac{1}{2}$.
(a) Write down the range of f .

A function g is such that $\mathrm{g}(x)=5 x-7, \quad$ for $x \in \mathbb{R}$.
(b) Find the exact solution of the equation $\operatorname{gf}(x)=13$.
(c) Find the solution of the equation $\mathrm{f}^{\prime}(x)=\mathrm{g}^{-1}(x)$.

4 (a)


The diagram shows the graph of $y=|\mathrm{f}(x)|$, where $\mathrm{f}(x)$ is a cubic. Find the possible expressions for $\mathrm{f}(x)$.
(b) (i) On the axes below, sketch the graph of $y=|2 x+1|$ and the graph of $y=|4(x-1)|$, stating the coordinates of the points where the graphs meet the coordinate axes.

(ii) Find the exact solutions of the equation $|2 x+1|=|4(x-1)|$.

5 (a) Find the vector which is in the opposite direction to $\binom{15}{-8}$ and has a magnitude of 8.5.
(b) Find the values of $a$ and $b$ such that $5\binom{3 a}{b}+\binom{2 a+1}{2}=6\binom{b+a}{2}$.

6 (a) Write down the values of $k$ for which the line $y=k$ is a tangent to the curve $y=4 \sin \left(x+\frac{\pi}{4}\right)+10$.
(b) (i) Show that $\frac{1+\tan \theta}{1-\cos \theta}+\frac{1-\tan \theta}{1+\cos \theta}=\frac{2(1+\sin \theta)}{\sin ^{2} \theta}$.
(ii) Hence solve the equation $\frac{1+\tan \theta}{1-\cos \theta}+\frac{1-\tan \theta}{1+\cos \theta}=3$, for $0^{\circ} \leqslant \theta \leqslant 360^{\circ}$.

7 (a) The first three terms of an arithmetic progression are $\lg 3,3 \lg 3,5 \lg 3$. Given that the sum to $n$ terms of this progression can be written as $256 \lg 81$, find the value of $n$.

## (b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

The first three terms of a geometric progression are $\ln 256, \ln 16, \ln 4$. Find the sum to infinity of this progression, giving your answer in the form $p \ln 2$.

## 8 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Find the exact coordinates of the points of intersection of the curve $y=x^{2}+2 \sqrt{5} x-20$ and the line $y=3 \sqrt{5} x+10$.
(b) It is given that $\tan \theta=\frac{\sqrt{3}-1}{2+\sqrt{3}}$, for $0<\theta<\frac{\pi}{2}$. Find $\operatorname{cosec}^{2} \theta$ in the form $a+b \sqrt{3}$, where $a$ and $b$ are constants.

9 A circle, centre $O$ and radius $r \mathrm{~cm}$, has a sector $O A B$ of fixed area $10 \mathrm{~cm}^{2}$. Angle $A O B$ is $\theta$ radians and the perimeter of the sector is $P \mathrm{~cm}$.
(a) Find an expression for $P$ in terms of $r$.
(b) Find the value of $r$ for which $P$ has a stationary value.
(c) Determine the nature of this stationary value.
(d) Find the value of $\theta$ at this stationary value.

10 The normal to the curve $y=\tan \left(3 x+\frac{\pi}{2}\right)$ at the point $P$ with coordinates $(p,-1)$, where $0<p \leqslant \frac{\pi}{6}$, meets the $x$-axis at the point $A$ and the $y$-axis at the point $B$. Find the exact coordinates of the mid-point of $A B$.

